$$(2)\ Ay^4+By^3-By-A=0,$$

the above test (K) does not hold. But if the equation be divided by A, it is seen as in §8 that $a=0$, so that y^2-1 is a factor of the reciprocal equation. The remaining factor is readily found.

V. If the quadratic equations are symmetrical,

$$a=b,\quad d=e,\quad a'=b',\quad d'=e'.$$

Equations (I) and (II) then become, after division by a and a',

$$x^2+y^2+c_2xy+d_1x+d_1y+f_1=0 \text{.....(I}'\text{)},$$

$$x^2+y^2+c_1xy+d_2x+d_2y+f_2=0 \text{.....(II}'\text{)}.$$

Subtracting, $$c_3xy+d_3x+d_3y+f_3=0 \text{.....(III}'\text{)}.$$

Substituting $x=\mu+\nu$, $y=\mu-\nu$ in (I′) and (III′),

$$a_1\mu^2+b_1\nu^2+2d_1\mu+f_1=0 \text{.....(IV}'\text{)},$$

$$c_3\mu^2-c_3\nu^2+2d_3\mu+f_3=0 \text{.....(V}'\text{)}.$$

From (IV′) and (V′), ν^2 may be eliminated, and the resulting quadratic in μ readily solved. Or the values for a, a', etc., may be substituted in (III). In this case the terms in ν^3 and ν will vanish and the resulting quartic in ν comes under the special case III just discussed.

AN EXTENSION TO CENTRAL CONICOIDS OF A THEOREM CONCERNING THE SEGMENT OF A SPHERE.

By G. W. GREENWOOD, McKendree College.

Consider the sphere, the cylinder, and the cone whose equations are, respectively,

$$x^2+y^2+z^2=1 \text{.....(1)},$$

$$x^2+y^2=1 \text{.....(2)},$$

$$x^2+y^2=z^2 \text{.....(3)}.$$

It is shown in text books that the volume of (1) included between planes parallel to the plane of x, y is equal to the volume of the segment of (2) diminished by the that of the segment of (3) included between the same planes.

If we employ the equations

$$x^2/a^2+y^2/b^2+z^2/c^2=1 \text{.....}(4),$$

$$x^2/a^2+y^2/b^2=1 \text{.....}(5),$$

$$x^2/a^2+y^2/b^2=z^2/c^2 \text{.....}(6),$$

for (1), (2), (3), respectively, the theorem is still true. If for (4) we substitute either of the equations

$$x^2/a^2+y^2/b^2=1+z^2/c^2 \text{.....}(7),$$

$$x^2/a^2+y^2/b^2=z^2/c^2-1 \text{.....}(8),$$

we get the theorems that the volume of a segment of (7) made by planes parallel to that of x, y is equal to the sum of the volumes of the corresponding segments of (5) and (6); and that of a segment of (7) is the volume of the corresponding segment of (6) diminished by that of the segment of (5).

The volume of a segment of (1) made by planes parallel to that of x, y is equal to the sum of the volumes of two cylinders whose bases are the bases of the segment and altitudes half that of the segment, together with the volume of a sphere to which the bases of the segment are tangent.

The corresponding theorems are as follows: The volume of a segment of (4) made by planes parallel to that of x, y is equal to the sum of the volumes of two cylinders whose bases are the bases of the segment and whose altitudes are half that of the segment, together with the volume of an ellipsoid to which the bases of the segment are tangent, similar to (4) and similarly placed.

Furthermore, the volume of a segment of (7) or (8) made by planes parallel to that of x, y is equal to the volumes of two cylinders whose bases are the bases of the segment, diminished by the volume of an ellipsoid to which the bases are tangent, whose axes are proportional to those of (7) or (8) and respectively parallel to them.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

178. **Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.**

A_n being the arithmetic mean of the nth powers of the numbers less than p and prime to it, find a relation between A_3, A_2 and p.